K25P 1897

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.) Examination, April 2025
(2023 and 2024 Admissions)
MATHEMATICS
MSMAT02C06/MSMAF02C06 : Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Give an example of a principal ideal domain. Justify your claim.

2. Find all the units in $\mathbb{Z}\left[\sqrt{-5}\right]$.

3. Prove that $\sqrt{\sqrt[3]{2}-i}$ is an algebraic number.

4. If α and β are constructible real numbers, prove that $\alpha\beta$ is also constructible.

5. Find the degree and a basis for the extension $\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)$ of $\mathbb{Q}$.

6. If $f(x) \in \mathbb{Q}[x]$ is irreducible over $\mathbb{Q}$, prove that all zeros of f(x) have multiplicity one. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. a) Prove that 5 is not an irreducible as an element in $\mathbb{Z}[i]$.

   b) Give an example to show that an integral domain can contain irreducibles that are not prime.

8. State Kronecker's theorem. Find an extension of $\mathbb{Q}$ containing a zero of $x^4 - 5x^2 + 6$.

P.T.O.

9. Prove that if F is a finite field, then for every positive integer n, there is an irreducible polynomial of degree n in F[x].

10. Prove that the field $\mathbb{Q}(\sqrt[3]{2})$ is not a splitting field over $\mathbb{Q}$.

11. Give an example of an irreducible polynomial over a field F, having a zero of multiplicity greater than one. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) State and prove Gauss's Lemma. **8**

    b) Prove that every Euclidean domain is a PID. **5**

13. a) If E is a finite extension field of a field F and K is a finite extension field of E, then prove that K is a finite extension of F. **8**

    b) State and prove the Fundamental Theorem of Algebra. **5**

14. a) Prove that trisecting an angle is impossible. **5**

    b) If F is a field of prime characteristic p with algebraic closure $\overline{F}$, then prove that $x^{p^n} - x$ has p^n distinct zeros in $\overline{F}$. **8**

15. a) State the Conjugation Isomorphism Theorem. **3**

    b) Let E be a finite extension of F. Then prove that E is a splitting field over F if and only if for every field extension K over E and for every isomorphism σ that fixes all the extensions of F and maps E onto a subfield of K, σ is an automorphism of E. **10**

16. State and prove primitive element theorem. **13**

(3×13=39)