



K25P 1897

Reg. No. :

Name :

II Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.)
Examination, April 2025
(2023 and 2024 Admissions)
MATHEMATICS

MSMAT02C06/MSMAF02C06 : Advanced Abstract Algebra

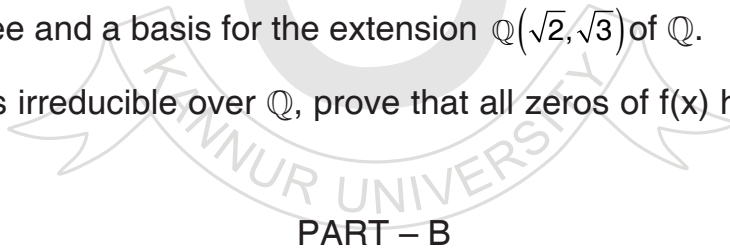
Time : 3 Hours

Max. Marks : 80



Answer **any five** questions. **Each** question carries **4** marks.

1. Give an example of a principal ideal domain. Justify your claim.
2. Find all the units in $\mathbb{Z}[\sqrt{-5}]$.
3. Prove that $\sqrt[3]{2-i}$ is an algebraic number.
4. If α and β are constructible real numbers, prove that $\alpha\beta$ is also constructible.
5. Find the degree and a basis for the extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ of $\mathbb{Q}$.
6. If $f(x) \in \mathbb{Q}[x]$ is irreducible over $\mathbb{Q}$, prove that all zeros of $f(x)$ have multiplicity one. **(5×4=20)**



Answer **any three** questions. **Each** question carries **7** marks.

7. a) Prove that 5 is not an irreducible as an element in $\mathbb{Z}[i]$.
b) Give an example to show that an integral domain can contain irreducibles that are not prime.
8. State Kronecker's theorem. Find an extension of $\mathbb{Q}$ containing a zero of $x^4 - 5x^2 + 6$.

P.T.O.



9. Prove that if F is a finite field, then for every positive integer n , there is an irreducible polynomial of degree n in $F[x]$.
10. Prove that the field $\mathbb{Q}(\sqrt[3]{2})$ is not a splitting field over $\mathbb{Q}$.
11. Give an example of an irreducible polynomial over a field F , having a zero of multiplicity greater than one. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) State and prove Gauss's Lemma. **8**
 b) Prove that every Euclidean domain is a PID. **5**
13. a) If E is a finite extension field of a field F and K is a finite extension field of E , then prove that K is a finite extension of F . **8**
 b) State and prove the Fundamental Theorem of Algebra. **5**
14. a) Prove that trisecting an angle is impossible. **5**
 b) If F is a field of prime characteristic p with algebraic closure $\bar{F}$, then prove that $x^{p^n} - x$ has p^n distinct zeros in $\bar{F}$. **8**
15. a) State the Conjugation Isomorphism Theorem. **3**
 b) Let E be a finite extension of F . Then prove that E is a splitting field over F if and only if for every field extension K over E and for every isomorphism σ that fixes all the extensions of F and maps E onto a subfield of K , σ is an automorphism of E . **10**
16. State and prove primitive element theorem. **13**
(3×13=39)
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